Technical Notes

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AIAA 82-4160

Analysis of a Characteristic of Laminar Corner Flow

Yasumasa Nomura*
National Defence Academy, Yokosuka, Japan

Introduction

ONE of the remarkable characteristics of laminar corner flow is the special shapes of its isovels in each section, normal to the corner line. Each isovel in the outer part of the boundary layer has an outward bulge on the plane of symmetry as shown in Fig. 1, which was discovered experimentally by many authors.[3-8,11] However, many researchers[1,2,9,10] theoretically predicted that isovels in the boundary layer always resemble hyperbolas in shape, and the outward bulge seems to have been generated due to some special reasons, such as the influence of the leading-edge profiles of both flat plates or lack of uniformity of the freestream velocity, etc.

This analysis was undertaken to determine whether these predictions are correct or not.

Basic Equations

Referring to the orthogonal curvilinear coordinates, the Navier-Stokes equations and the equation of continuity can be written as follows[12]:

$$u\frac{\partial u}{\partial x}+\frac{v}{H}\frac{\partial u}{\partial \eta}+\frac{w}{H}\frac{\partial u}{\partial \zeta}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\nabla^2 u \tag{1}$$

$$u\frac{\partial v}{\partial x}+\frac{v}{H}\frac{\partial v}{\partial \eta}+\frac{w}{H}\frac{\partial v}{\partial \zeta}+\frac{vw}{H^2}\frac{\partial H}{\partial \zeta}-\frac{w^2}{H^2}\frac{\partial H}{\partial \eta}$$
$$=-\frac{1}{\rho H}\frac{\partial p}{\partial \eta}-\frac{\nu}{H}\left[\frac{\partial}{\partial x}\left\{\frac{\partial u}{\partial \eta}-\frac{\partial (Hv)}{\partial x}\right\}\right.$$
$$\left.-\frac{\partial}{\partial \zeta}\left\{\frac{1}{H^2}\frac{\partial (Hv)}{\partial \zeta}-\frac{1}{H^2}\frac{\partial (Hw)}{\partial \eta}\right\}\right] \tag{2}$$

$$u\frac{\partial w}{\partial x}+\frac{v}{H}\frac{\partial w}{\partial \eta}+\frac{w}{H}\frac{\partial w}{\partial \zeta}+\frac{vw}{H^2}\frac{\partial H}{\partial \eta}-\frac{v^2}{H^2}\frac{\partial H}{\partial \zeta}$$
$$=-\frac{1}{\rho H}\frac{\partial p}{\partial \zeta}-\frac{\nu}{H}\left[\frac{\partial}{\partial \eta}\left\{\frac{1}{H^2}\frac{\partial (Hv)}{\partial \zeta}-\frac{1}{H^2}\frac{\partial (Hw)}{\partial \eta}\right\}\right.$$
$$\left.-\frac{\partial}{\partial x}\left\{\frac{\partial (Hw)}{\partial x}-\frac{\partial u}{\partial \zeta}\right\}\right] \tag{3}$$

$$\frac{\partial (H^2 u)}{\partial x}+\frac{\partial (Hv)}{\partial \eta}+\frac{\partial (Hw)}{\partial \zeta}=0 \tag{4}$$

Received March 12, 1981; revision received Aug. 10, 1981.

*Professor, Dept. of Aeronautical Engineering. Member AIAA.

where x represents the streamwise distance from the leading edge of plate A. η, ζ, and H are defined by the following equations,

$$\eta=r^{1/n}\sin(\theta/n),\quad \zeta=r^{1/n}\cos(\theta/n),\quad H=nr^{(n-1)/n},\quad n=\alpha/\pi$$

and u, v, and w are the velocity components in the x, η, and ζ direction, respectively. $\alpha(\pi>\alpha>0)$ is the angle between plates A and B, which are shown in Fig. 2.

Approximate Equation in the Neighborhood of the Plane of Symmetry

Since ζ is small in the neighborhood of the plane of symmetry, H can be written as

$$H\simeq n\eta^{(n-1)}e^{(n-1)\bar{\zeta}^2/2}$$

Now, we put

$$u=Uf'(Y^q,\bar{\zeta})e^{(1-n)\bar{\zeta}^2},\; w=\sqrt{\frac{\nu U}{x}}\,\bar{\zeta}h'(Y^q,\bar{\zeta})e^{(1-n)\bar{\zeta}^2/2},\; \frac{\partial p}{\partial x}=0$$

where $Y=(U/\nu x)^{1/2n}\eta$ and $\bar{\zeta}=\zeta/\eta$ where U is a freestream velocity and the prime indicates differentiation with respect to Y^q and q is a constant. Substituting them into Eqs. (1) and (4), and assuming that the two-dimensional boundary-layer approximation can be extended to this three-dimensional case

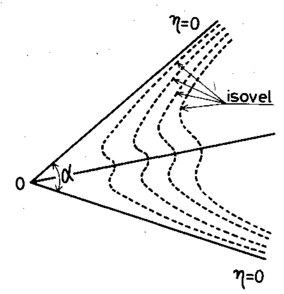

Fig. 1 Isovels in the boundary layer.

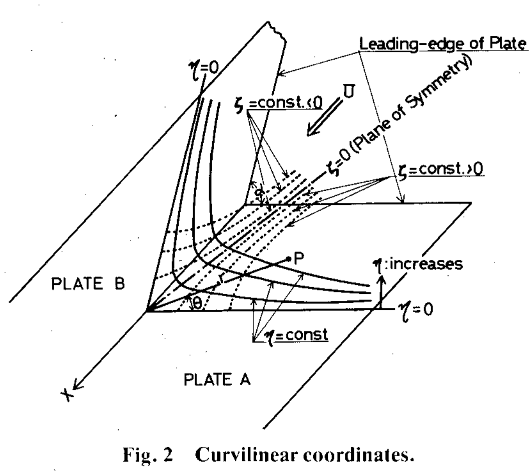

Fig. 2 Curvilinear coordinates.

such that

$$u \gg v, \quad u \gg w, \quad \frac{\partial u}{\partial \eta} \gg \frac{\partial u}{\partial x}, \quad \frac{\partial u}{\partial \zeta} \gg \frac{\partial u}{\partial x}$$

etc., then the following equation can be obtained in the neighborhood of the plane of symmetry:

$$-\frac{n}{2} Y^{(2n+q-2)} f' f'' + \frac{n}{2} Y^{(q-1)} f'' \int_A^{Y^q} \Big\{ Y^{(2n-1)} f'' - \frac{2}{q} Y^{(n-q-1)} h' \Big\} \mathrm{d}(Y^q) = q Y^{(2q-2)} f''' + (q-1) Y^{(q-2)} f'' + \Big\{ \frac{2(1-n)}{q} f' + \frac{1}{q} f'_{\zeta\zeta} \Big\} Y^{-2} \tag{5}$$

In this domain, we can also write

$$v = \frac{1}{2}\sqrt{\frac{\nu U}{x}}\, Y^{(1-n)} e^{(1-n)\zeta^2/2} \int_A^{Y^q} \Big\{ Y^{(2n-1)} f'' - \frac{2}{q} Y^{(n-q-1)} h' \Big\} \mathrm{d}(Y^q) \tag{6}$$

Since $\lim_{Y\to\infty} v \neq 0$, it can be decided that $A=\infty$ when $q>0$ and $A=0$ when $q<0$.

Isovels in the Neighborhood of the Plane of Symmetry (Case of $\alpha = 90$ deg)

When α is 90 deg, we use $q=1$. Then, isovels in the neighborhood of the plane of symmetry can be written as follows:

$$\text{const} = u/U = f' \exp[\tfrac{1}{2}\cot^2(2\theta)] \tag{7}$$

Differentiating this twice by θ and putting $\theta = 45$ deg and $\bar{r}_\theta = 0$, we get

$$\bar{r}^2 - \bar{r}\bar{r}_{\theta\theta} = \frac{2(f' + f'_{\zeta\zeta}) - \bar{r}^2 f''}{f''} \tag{8}$$

where $\bar{r} = \sqrt{U/\nu x}\, r$ and the prime means differentiation with respect to Y.

On the other hand, putting $n = \tfrac{1}{2}$ and $q=1$ into Eq. (5), we get

$$\frac{Y^2}{4} f''(-1-g) - Y^2 f''' - F = 0 \tag{9}$$

where

$$g = 2\int_\infty^Y Y^{(-3/2)} h' \,\mathrm{d}Y, \quad F = f' + f'_{\zeta\zeta}$$

Solving Eq. (9), we get

$$f'' = \exp[-\tfrac{1}{4}\textstyle\int(1+g)\,\mathrm{d}Y] \displaystyle\int_\infty^Y (-F/Y^2) \exp[\tfrac{1}{4}\textstyle\int(1+g)\,\mathrm{d}Y]\,\mathrm{d}Y \tag{10}$$

In the vicinity of the outer edge of the boundary layer, one can assume that $f' \simeq 1$ and $f'_{\zeta\zeta} \simeq 0$; consequently, $F \simeq 1$. From this, we can find that

$$\int_\infty^Y (-F/Y^2) \exp[\tfrac{1}{4}\textstyle\int(1+g)\,\mathrm{d}Y]\,\mathrm{d}Y > 0$$

i.e., $f'' > 0$.

On the other hand, since $Hv = (U/4x)(f' - 1 - g)$, we can get $\lim_{Y\to\infty} g = 0$, which gives

$$\lim_{\bar{r}\to\infty} \bar{r}^2 f'' = \lim_{Y\to\infty} Y f'' = \lim_{Y\to\infty} \frac{F}{(1+g)Y/4 + 1} = 0 \tag{11}$$

Therefore, substituting these into Eq. (8) yields

$$\bar{r}^2 - \bar{r}\bar{r}_{\theta\theta} > 0 \tag{12}$$

This shows that each isovel has an outward bulge on the plane of symmetry.

Isovels in the Neighborhood of the Plane of Symmetry (Case of 180 deg $> \alpha > 0$ deg, $\alpha \neq 90$ deg)

In this case, we put $q = 2n-1$. Then each isovel in the neighborhood of the plane of symmetry can be written as follows:

$$\text{const} = u/U = f' \exp[(1-n)\cot^2(\theta/n)] \tag{13}$$

Differentiating this twice with respect to θ and putting $\theta = n\pi/2$, $\bar{r}_\theta = 0$, we get the following:

$$\bar{r}^2 - \bar{r}\bar{r}_{\theta\theta} = \frac{r^{1/n}\{F_1 + (n-1)(2n-1)\bar{r}^{(2-1/n)} f''\}}{n(2n-1)f''} \tag{14}$$

where the prime means differentiation with respect to $Y^{(2n-1)}$ and $F_1 = 2(1-n)f' + f'_{\zeta\zeta}$. Then, putting $q = 2n-1$ and $f = Y^{(2n-1)}$ in Eq. (5),

$$f''' + \frac{n/2Y - 2(1-n)Y^{(1-2n)} + GY^{(2-2n)}}{(2n-1)} f'' = -\frac{F_1 Y^{(2-4n)}}{(2n-1)^2} \tag{15}$$

where

$$G = \int_A^{Y^{(2n-1)}} \frac{n}{2n-1} Y^{-n} h' \,\mathrm{d}Y^{(2n-1)}$$

From Eq. (15),

$$f'' = -\exp\Big[-\int \frac{n/2Y - 2(1-n)Y^{(1-2n)} + GY^{(2-2n)}}{(2n-1)} \mathrm{d}Y^{(2n-1)}\Big] \times \int_A^{Y^{(2n-1)}} \Big[\frac{F_1 Y^{(2-4n)}}{(2n-1)^2} \times \exp\Big\{\int \frac{n/2Y - 2(1-n)Y^{(1-2n)} + GY^{(2-2n)}}{(2n-1)} \mathrm{d}Y^{(2n-1)}\Big\}\Big] \mathrm{d}Y^{(2n-1)} \tag{16}$$

In the vicinity of the outer edge of the boundary layer, we get $F_1 \simeq 2(1-n) > 0$. Substituting this into Eq. (16), it is found that $f'' > 0$ when $n > \tfrac{1}{2}$ and $f'' < 0$ when $n < \tfrac{1}{2}$.

On the other hand, we obtain

$$\lim_{\bar{r}\to\infty} \bar{r}^{(2-1/n)} f'' = \lim_{Y\to\infty} \Big\{ \frac{F_1}{(2n-1)(nY^{2n}/2 + GY - 1)} \Big\} = 0 \tag{17}$$

Then, substituting these into Eq. (14) yields

$$\bar{r}^2 - \bar{r}\bar{r}_{\theta\theta} > 0 \tag{18}$$

This shows that each isovel has an outward bulge also in this case.

Conclusions

The analysis proves that each isovel in the vicinity of the outer edge of the laminar boundary layer has an outward bulge on the plane of symmetry for any value of $\alpha(\pi>\alpha>0)$ without any special conditions.

References

[1]Loiziansky, L. G. and Bolshakov, V. P., "On Motion of Fluid in Boundary-Layer near Line of Intersection of Two Planes," NACA TM 1308, 1936.

[2]Carrier, G. F., "The Boundary Layer in a Corner," *Quarterly of Applied Mathematics*, Vol. 4, No. 4, 1946, pp. 367-370.

[3]Nomura, Y., "Theoretical and Experimental Investigations on the Incompressible Viscous Flow around the Corner," *Memoirs of the Defence Academy of Japan*, Vol. 2, No. 3, 1962, pp. 117-148.

[4]Rubin, S. G. and Grossman, B., "Viscous Flow along a Corner. Pt. 2. Numerical Solution of Corner-Layer Equations," PIBAL Rept. No. 69-33, 1969.

[5]Tokuda, N., "Viscous Flow near a Corner in Three Dimensions," *Journal of Fluid Mechanics*, Vol. 53, Pt. 1, 1972, pp. 129-148.

[6]Zamir, M. and Young, A. D., "Experimental Investigation of the Boundary Layer in a Streamwise Corner," *Aeronautical Quarterly*, Vol. 21, Pt. 4, 1970, pp. 313-338.

[7]Zamir, M., "On the Corner Boundary Layer with Favourable Pressure Gradient," *Aeronautical Quarterly*, Vol. 23, Pt. 2, 1972, pp. 161-168.

[8]Zamir, M., "Further Solution of the Corner Boundary-Layer Equations," *Aeronautical Quarterly*, Vol. 24, Pt. 3, 1973, pp. 219-226.

[9]Barclay, W. H., "Experimental Investigation of the Laminar Flow along a Straight 135° Corner," *Aeronautical Quarterly*, Vol. 24, Pt. 2, 1973, pp. 147-154.

[10]El-Gamal, H. A. and Barclay, W. H., "Experiments on the Laminar Flow in a Rectangular Streamwise Corner," *Aeronautical Quarterly*, Vol. 29, Pt. 2, 1978, pp. 75-97.

[11]Nomura, Y. and Terada, H., "On the Transition of Corner Boundary Layer," *Transactions of the Japanese Society of Aerospace Sciences*, Vol. 21, No. 53, 1978, pp. 111-117.

[12]Meksyn, D., *New Methods in Laminar Boundary-Layer Theory*, Pergamon Press, London, 1961, pp. 8-11.

AIAA 82-4161

A Hot-Film Static-Pressure Probe

George C. Ashby Jr.* and Leonard M. Weinstein†
NASA Langley Research Center, Hampton, Va.

Nomenclature

d = probe tube diameter, cm
ℓ = distance from probe tip to orifices, cm
p_{HF} = pressure measured by hot-film probe, mmHg
p_c = pressure measured by conventional probe, mmHg
$R_{c,l}$ = resistance of unheated sensor, Ω
R_{cold} = cold resistance of heated sensor, Ω
R_{hot} = resistance of heated sensor with 1.5 overheat ratio, Ω
N_{Re} = freestream unit Reynolds number, 1/cm
V_B = output of rear sensor, mV

Received Oct. 15, 1981; revision received Dec. 3, 1981. This paper is declared a work of the U.S. Government and therefore is in the public domain.

*Aero-Space Technologist, Vehicle Analysis Branch, Space Systems Division. Member AIAA.

†Aero-Space Technologist, Viscous Flow Branch, High Speed Aerodynamics Division.

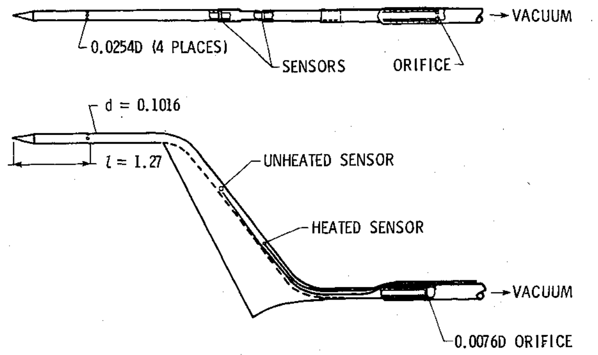

Fig. 1 Conventional probe configuration and the internal locations of the hot-film sensors and the sonic orifice (all dimensions in cm).

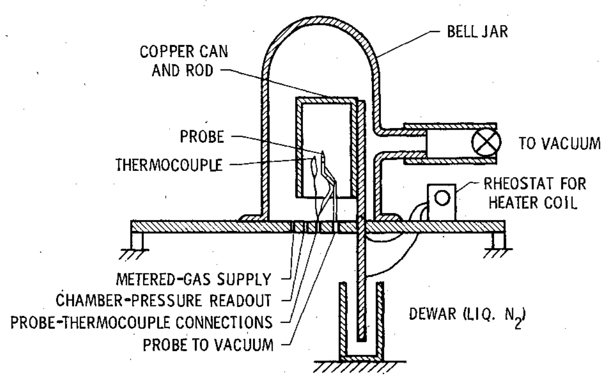

Fig. 2 Probe calibration apparatus.

Introduction

A CONICAL-NOSE, static-pressure probe with orifices located ten or more tube diameters downstream of the tip[1] can measure freestream pressure within 1-2% in supersonic flow. However, for blowdown tunnels, the pressure settling time in the connecting tubing and the readout equipment may be of the order of the tunnel run time. This makes static pressure surveys of the flowfield between a body and its shock impractical because of the large number of runs required to traverse the region.

To reduce response time, increase traverse speed, and obtain reasonably accurate values of static pressure, a probe has been developed which consists of a pair of hot-film sensors and a small orifice installed inside the probe downstream of the exterior orifices (Fig. 1). The rear sensor is operated at an overheat ratio ($R_{\text{hot}}/R_{\text{cold}}$) of 1.5 to 1.8. A constant-temperature anemometer keeps the sensor at the temperature set by the overheat ratio, and the bridge voltage will vary according to the velocity, pressure, and temperature of the fluid. By installing a sonic orifice downstream of the sensors and connecting the probe to a vacuum pump, the velocity of the fluid over the sensors will be constant and low subsonic. To account for the fluid temperature, a second sensor (front) is operated as a resistance thermometer (at a low-fixed current).

The procedure is to calibrate the probe in the gas of the interest over the range of temperatures and pressures anticipated in the wind-tunnel tests and then apply the calibration to reduce the data from those tests. Some other details of this study are contained in a paper[2] published previously.

The concept of using hot-film sensors and a sonic orifice is similar to that of Remenyik and Kovasznay,[3] who used a single hot wire in a wall orifice to measure the pressure fluctuations, and to that of Thermo-Systems Inc., who use a single sensor and a sonic orifice in their aspirating probes.[4]